Remember that with the sine and cosine functions the period = $\frac{2\pi}{b}$ where b is the coefficient of the rgument.

When $0 < b < 1$, the period of $y = a\sin bx$ is greater than 2π and represents a horizontal stretching of the graph of $y = a\sin x$.

When $b > 1$, the period of $y = a\sin bx$ is less than 2π and represents a horizontal shrinking of the graph of $y = a\sin x$.

The constant c in the general equations $y = a\sin(bx - c)$ and $y = a\cos(bx - c)$ creates horizontal translations (shifts) of the basic sine and cosine curves.

The period of $y = a\sin(bx - c)$ is $\frac{2\pi}{b}$, and the graph of $y = a\sin bx$ is shifted by an amount $\frac{c}{b}$. The number $\frac{c}{b}$ is called the phase shift.

Graphs of Sine and Cosine Functions

The graphs of $y = a\sin(bx - c)$ and $y = a\cos(bx - c)$ have the following characteristics.

(Assume b > 0.)

Amplitude = $|a|$ Period = $\frac{2\pi}{b}$

The left and right endpoints of one-cycle interval can be determined by solving the equations $bx - c = 0$ and $bx - c = 2\pi$.

Horizontal Translations

b=1

Ex:4 a.) Sketch the graph of $y = 2\sin\left(1x - \frac{\pi}{2}\right)$ for one full cycle of output values (one period).

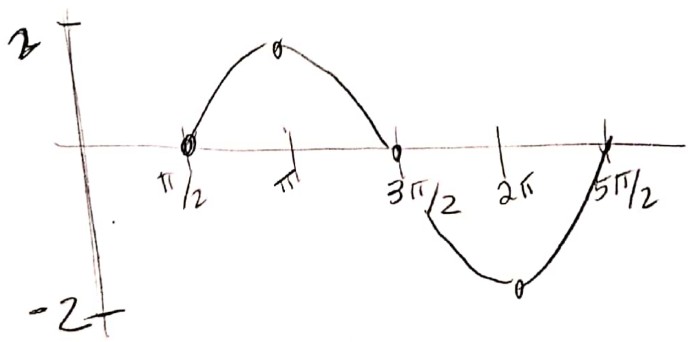
π

Label the key points on the graph.

Amp = 2 Per = 2π Phase Shift = $\frac{\pi}{2}$ ($\frac{c}{b}$)

left endpt $\frac{\pi}{2}$

right endpt $\frac{\pi}{2} + 2\pi = \frac{5\pi}{2}$



state pts

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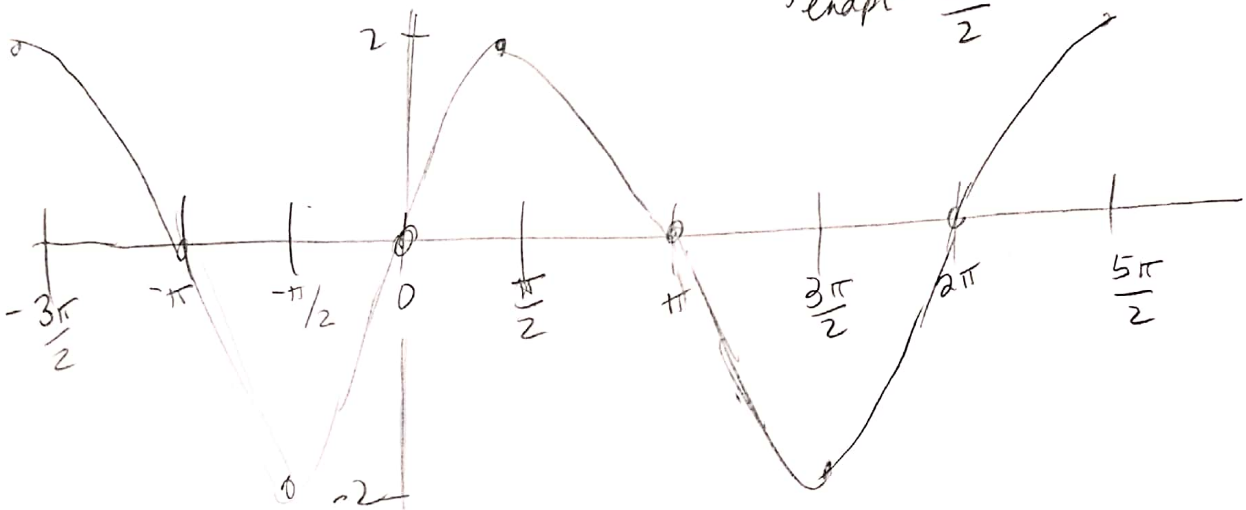
b.) Sketch the graph of $y = 2\cos\left(x - \frac{\pi}{2}\right)$ for two full cycles of output values (two periods).

Label the key points on the graph.

Amp = 2 Per = 2π Phase Shift = left endpt $\frac{\pi}{2}$

Right endpt $\frac{5\pi}{2}$



Ex:5 a.) State the amplitude, period and the phase shift of $y = 2\sin(3\pi x - 2\pi)$

Amp = 2 Per = $\frac{2\pi}{b}$ $\frac{2\pi}{3\pi} = \frac{2}{3}$ Phase Shift = $\frac{c}{b}$ $\frac{2\pi}{3\pi} = \frac{2}{3}$ ← left endpt

$\frac{4}{3}$ ← right endpt

b.) State the amplitude, period, and the phase shift of $y = -3\cos(2\pi x + 4\pi)$ $= 0$ $= 2\pi$

Amp = 3 Per = $\frac{2\pi}{2\pi} = 1$ Phase Shift = $-\frac{4\pi}{2\pi} = -2$ ← left endpt

-1 ← right endpt

− means reflect over x axis

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The final type of transformation is the vertical translation caused by the constant d in the equations

$y = d + a\sin(bx - c)$ and $y = d + a\cos(bx - c)$.

$a\sin(bx-c)+d$ $a\cos(bx-c)+d$

The shift d units up for $d > 0$ and d units down for $d < 0$. In other words, the graph oscillates about the horizontal line $y = d$ instead of about the x-axis.

<u>Vertical Translations</u> $y = -\cos 3x + 4$

<u>Ex:6</u> Sketch the graph of $y = 4 - \cos 3x$ for two full cycles of output values (two periods).

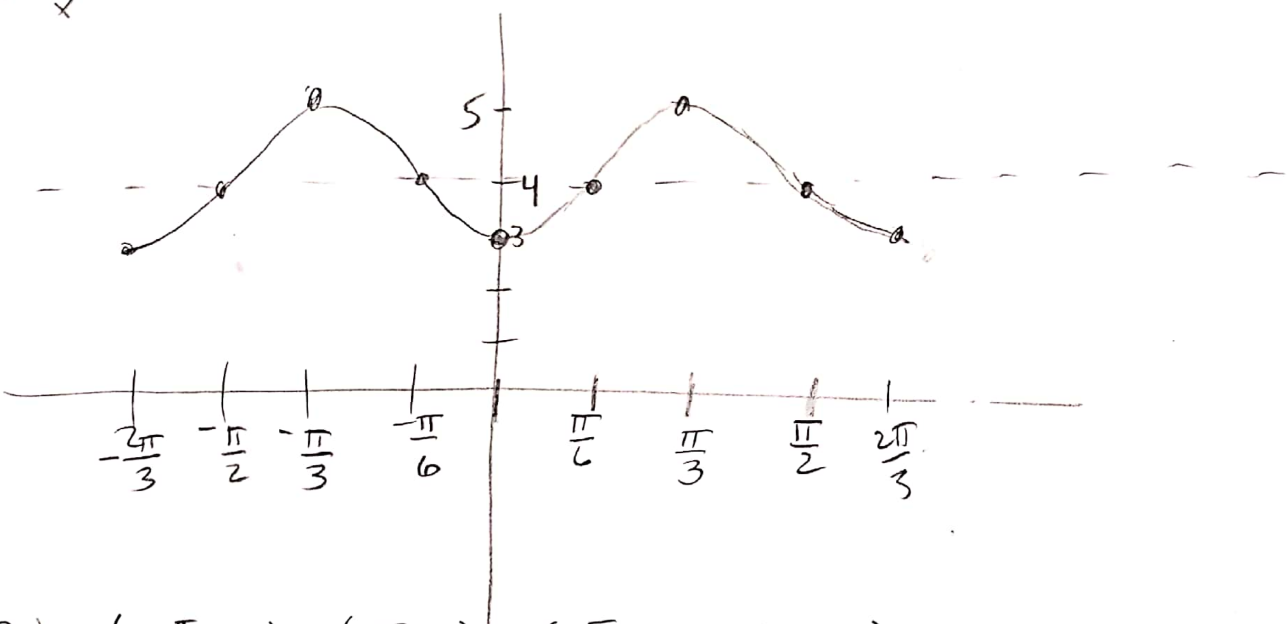
Label the key points on the graph. easier

Amp = 1 Per = $\frac{2\pi}{3}$ Phase Shift = $\varnothing$ Vertical Shift = 4

$0 \to \frac{2\pi}{3}$

1 per.

- means reflect over x



$(-\frac{2\pi}{3}, 3)$ $(-\frac{\pi}{2}, 4)$ $(-\frac{\pi}{3}, 5)$ $(-\frac{\pi}{6}, 4)$ $(0, 3)$

$(\frac{\pi}{6}, 4)$ $(\frac{\pi}{3}, 5)$ $(\frac{\pi}{2}, 4)$ $(\frac{2\pi}{3}, 3)$